Closing Tue: 13.3(part 2), 13.4
Closing Thu: 14.1, 14.3 (part 1)

14.1/14.3 Introduction to Multivariable Functions, Surfaces, and Partial Derivatives

Def'n: A function, *f*, of two variables is a rule that assigns a number for each input (x,y). We write

$$z = f(x,y)$$

We sometimes write $f: \mathbb{R}^2 \to \mathbb{R}$ to indicate that the inputs to *f* have 2-dimensions and the output has one dimension.

The set of allowable inputs is called the **domain**. The domain will be a region in 2-dimensions. The set of possible outputs is called the **range**.

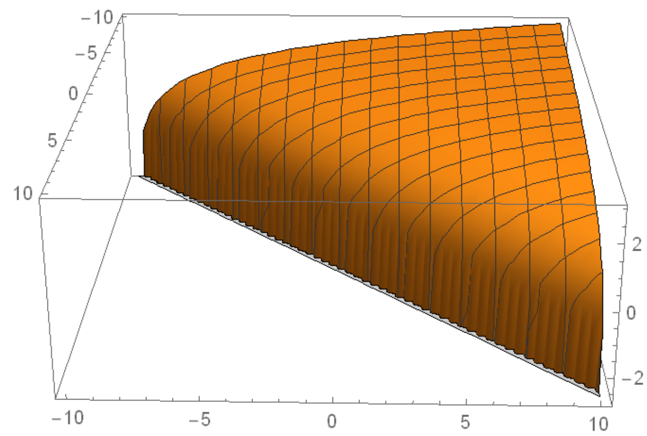
Domain:
Any question that asks "find the domain" is simply asking you if you know your functions well enough to understand when they are not defined.

Appears in the Function	*Restriction*
$\sqrt{BLAH}$	BLAH ≥ 0
STUFF/BLAH	BLAH $\neq 0$
ln(BLAH)	BLAH > 0
$\sin^{-1}$(BLAH)	$-1 \leq$ BLAH ≤ 1
and other trig…	

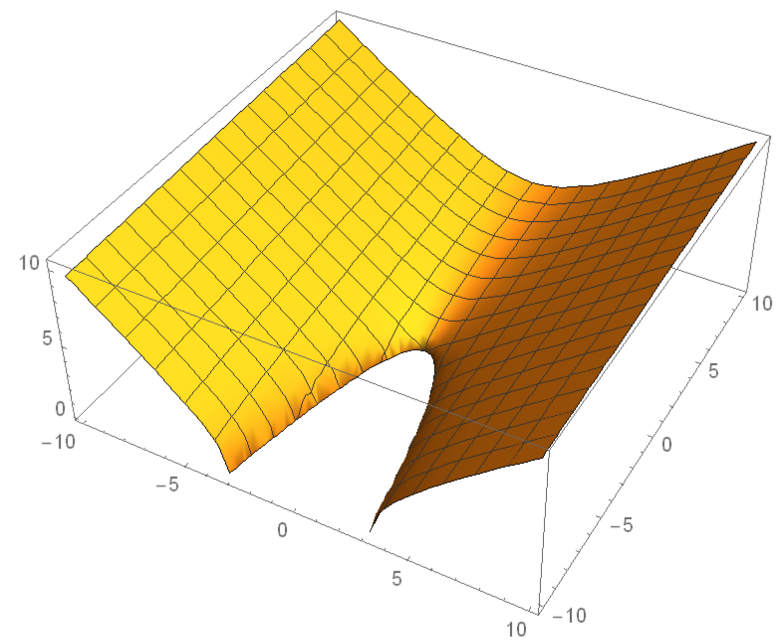
Examples:

Sketch the domain of

(1) $f(x,y) = \ln(y - x)$



(2) $g(x,y) = \sqrt{y + x^2}$



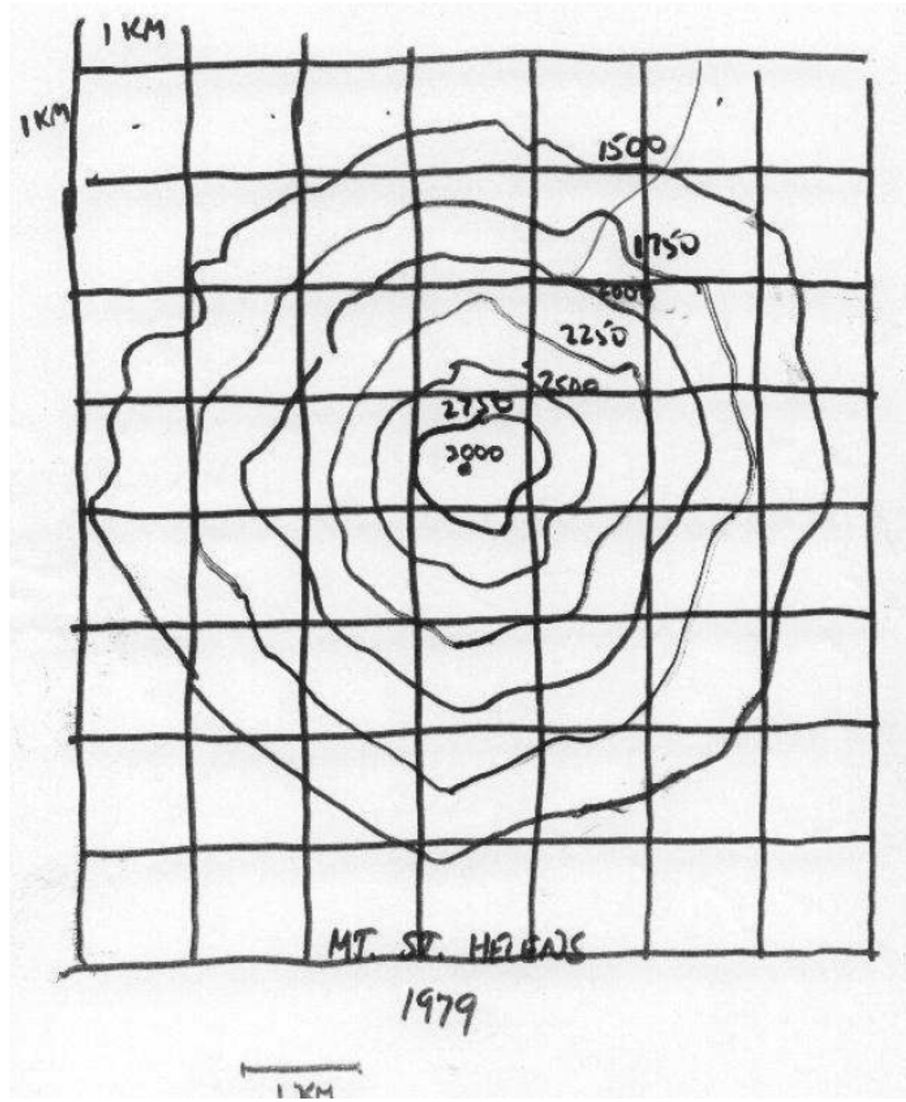
Visualizing Surfaces

The basic tool for graphing surfaces is **traces**. When z = f(x,y), we think of z as height and we typically look at traces given by fixed values of z first.

We call these traces **level curves**, because each curve represents all the points at the same height (level) on the surface.

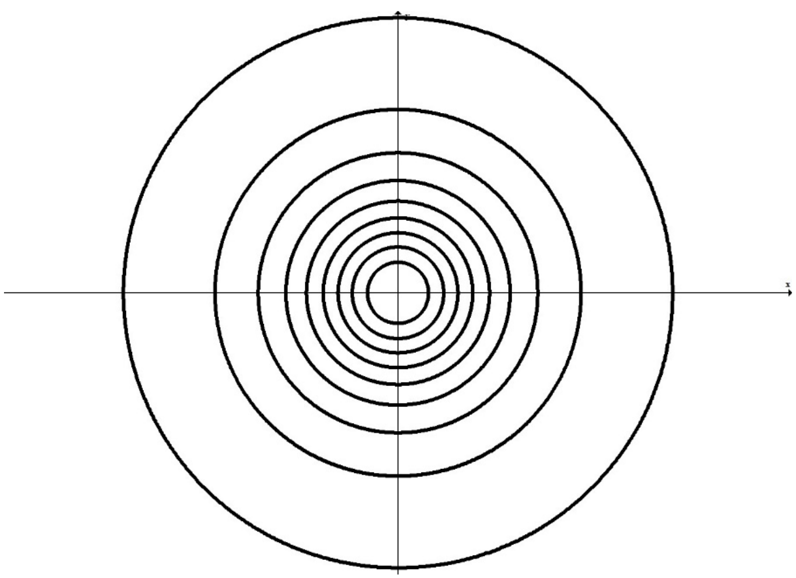
A collection of level curves is called a **contour map** (or **elevation map**).

Contour Map (Elevation Map) of
Mt. St. Helens from 1979 (before it erupted):

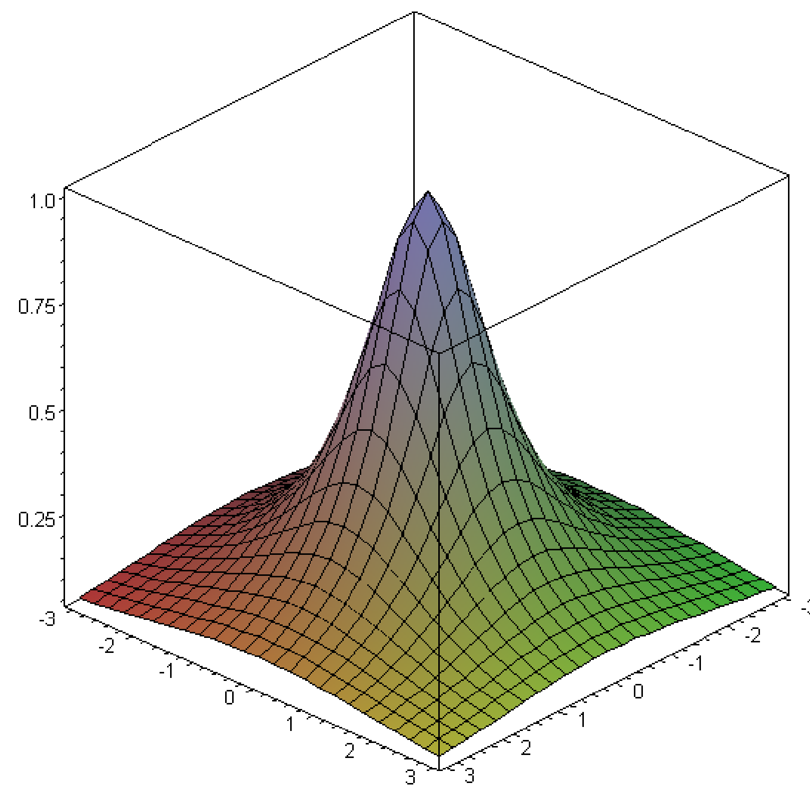


Examples:

Level Curves for $z = f(x, y) = \frac{1}{1+x^2+y^2}$
at z = 1/10, 2/10, … , 9/10, 10/10



Graph of $z = f(x, y) = \frac{1}{1+x^2+y^2}$



Graph of $z = \sin(x) - y$